Lecture # 23

2.17 STREAMLINES

A streamline is a curve drawn in the fluid such that the tangent to it at every point is in the direction of fluid velocity $\overrightarrow{V}$ at that point at the instant considered. A streamline is also called the line of flow.

Since a streamline is everywhere parallel to the direction of flow, we may conclude that there cannot be any flow across a streamline i.e. the fluid cannot cross a streamline. Thus physically, the streamlines are viewed as being solid boundaries with the fluid flowing with them.

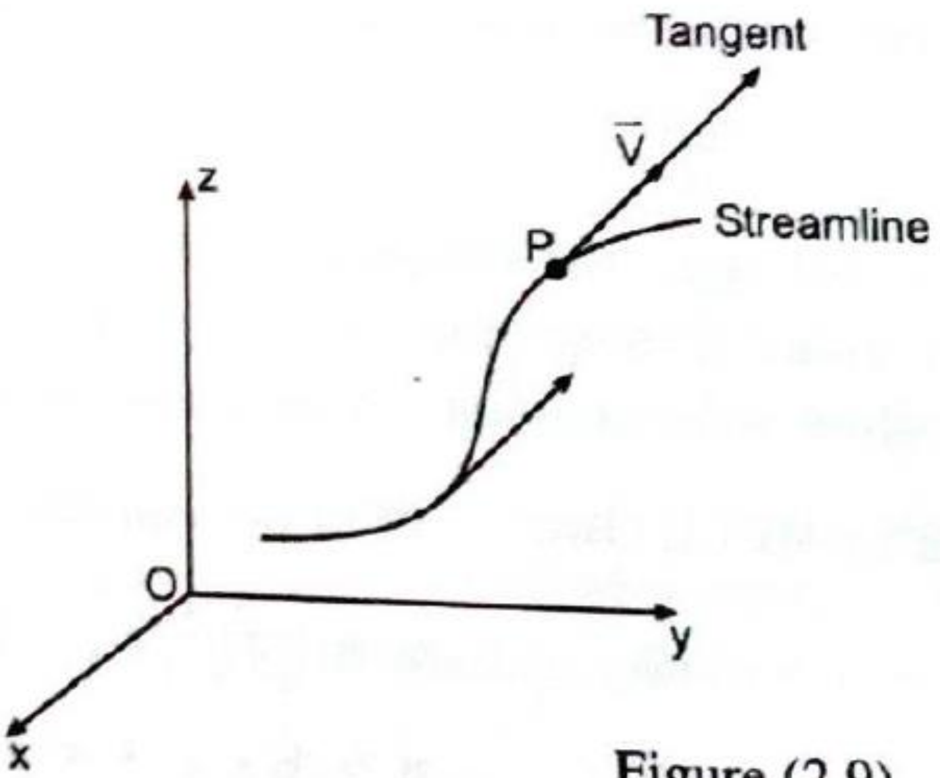

Figure (2.9)

When the motion is steady , so that the flow pattern does not change with time , the streamlines have the same shape at all instants . In other words , the streamlines are the fixed lines in the flow field at all times . However , when the motion is unsteady , the flow pattern changes with time and the streamlines will vary from instant to instant . The photographs taken at different instants will reveal a different system of streamlines . The whole set of streamlines at a given instant , thus gives the flow pattern at that instant .

NOTE: It should be noted that the boundaries of stationary solid surfaces are always streamlines since the fluid cannot cross solid boundaries .

DIFFERENTIAL EQUATION FOR THE STREAMLINES

Since at each point of a streamline, the velocity vector $\vec{V}$ is parallel to the unit tangent vector $\hat{t} = \frac{d\vec{r}}{ds}$ to the streamline at that point, therefore $\vec{V} \times \frac{d\vec{r}}{ds} = \vec{0}$ or $\vec{V} \times d\vec{r} = \vec{0}$

i.e. $$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & v & w \\ dx & dy & dz \end{vmatrix} = \vec{0}$$

or $$(v\,dz - w\,dy)\,\hat{i} + (w\,dx - u\,dz)\,\hat{j} + (u\,dy - v\,dx)\,\hat{k} = \vec{0}$$

or $$v\,dz - w\,dy = 0 \qquad (1)$$

$$w\,dx - u\,dz = 0 \qquad (2)$$

$$u\,dy - v\,dx = 0 \qquad (3)$$

From equations (1) , (2) , and (3) , we get

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad (4)$$

Equation (4) is the differential equation for the streamlines regardless whether the flow is steady or unsteady .

This equation will have a pair of differential equations , which when integrated will give two equations for the streamlines , each containing a constant of integration . The streamlines are the curves of intersection of these two equations thus obtained . Thus equation (4) has a doubly infinite set of solutions corresponding to different values of the constants . The particular values of these constants serve to identify a streamline .

For two dimensional flow , the differential equation (4) reduces to

$$\frac{dx}{u} = \frac{dy}{v} \tag{5}$$

In this case , the streamlines will be obtained by integrating equation (5) . Since there will be only one constant of integration , we will have an infinite number of different streamlines corresponding to different values of the constant . A particular value of the constant serves to identify the streamline .

EXAMPLE (10): Find the equations of streamlines for the following flow fields :

(i) $u = \dfrac{k\,x}{x^2 + y^2}\ , \quad v = \dfrac{k\,y}{x^2 + y^2}$

(ii) $u = k\,x\ , \quad v = -k\,y$ (k constant) .

SOLUTION: (i) Since the velocity field is steady and two – dimensional , therefore , from equation (5)

$$\frac{d\,x}{\dfrac{k\,x}{x^2 + y^2}} = \frac{d\,y}{\dfrac{k\,y}{x^2 + y^2}} \quad \text{or} \quad \frac{d\,x}{x} = \frac{d\,y}{y}$$

which on integration gives $\ln y = \ln x + \ln C$ or $y = C\,x$ where C is the constant of integration . Hence , the streamlines are the straight lines radiating from the origin as shown in the figure (2.10) .

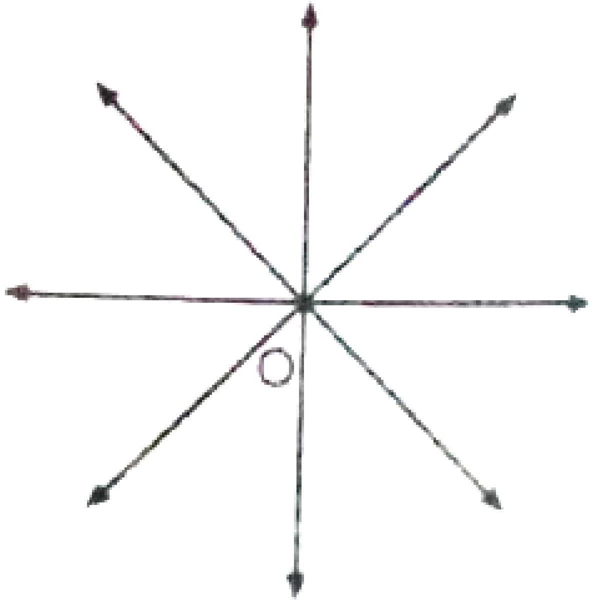

Figure (2.10)

(ii) Since the flow field is steady and two – dimensional , therefore , from equation (5) we have

$$\frac{d\,x}{k\,x} = \frac{d\,y}{-k\,y} \quad \text{or} \quad \frac{d\,x}{x} + \frac{d\,y}{y} = 0$$

or $\ln x + \ln y = \ln C$ or $x\,y = C$

... of integration .

This is the general expression for the streamlines , which are rectangular hyperbolas . The complete pattern is plotted by assigning various values to the constant C . The arrowheads can be determined only by returning back to the given velocity field to ascertain the velocity – component directions . For example , in the first quadrant ($x > 0$, $y > 0$) , u is positive and v is negative ; hence the flow moves down and to the right establishing the arrow heads as shown in figure (2.11) . Finally , note the peculiarity that the two streamlines ($C = 0$) have opposite directions and intersect each other . This is possible only at a point where $u = v = 0$, which occurs at the origin in this case . Such a point of zero velocity is called a **stagnation** point .

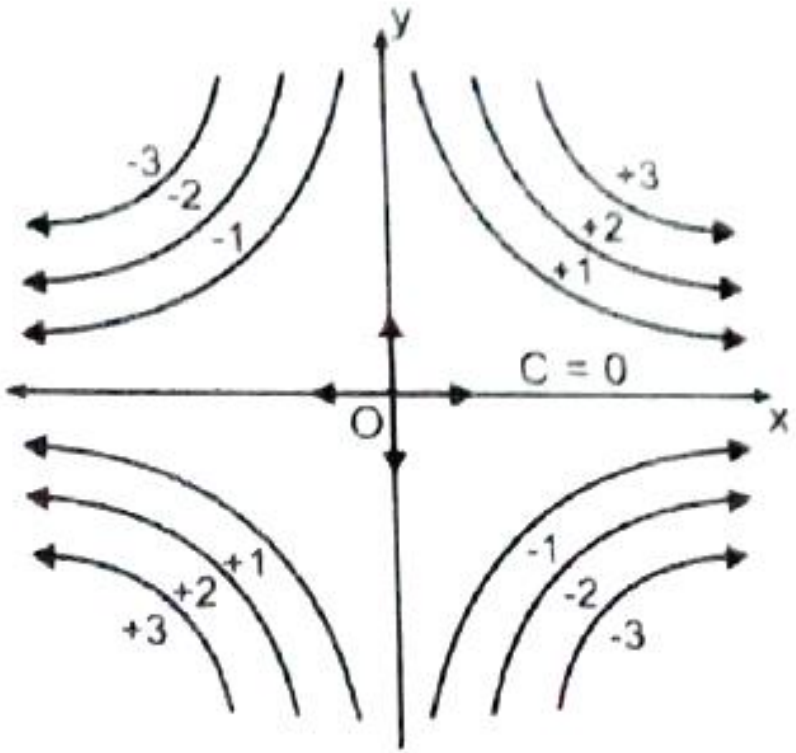

Figure (2.11)